The Impact of Reserve Prices on the Technology of Procured Items

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ABSTRACT

Although, Federal and Defense Acquisition Regulations permit "no-awards", the federal government very rarely employs an optimal reserve pricing policy when procuring commodities by sealed bid auction. It is an obvious implication from the auction literature that an optimal reserve policy would increase expected surplus for the government. In this paper we demonstrate two other consequences associated with the use of an optimal reserve. First, if the procurement official must select a minimum technology level for the items to be procured then, under fairly reasonable conditions, the use of an optimal reserve will result in an increase in the technology level of the procured items. Second, when a procurement official suffers from a particular form of agency problem which we refer to as a "technology bias" we demonstrate that preventing him from using an optimal reserve policy may result in greater expected surplus for taxpayers. Intuitively, if the procurement official is more concerned with technology than with cost then the reserve policy exacerbates the bias toward high technology items that are not surplus maximizing (compared to their low technology counterparts). Consequently, acquisition regulations that inhibit the use of an "optimal" reserve are, under certain circumstances, socially beneficial. However, if the government can effectively monitor procurement officials in order to dampen or eliminate the agency problem then an optimal reserve policy will be beneficial to both taxpayers (higher expected surplus) and to procurement officials (via procurement of higher technology levels).

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Reserve prices have played an important role in the auction literature. In many circumstances a revenue maximizing auctioneer can achieve his objective through the optimal choice of a reserve. Although other strategies are available, such as entry fees, reserves are of particular interest due to their widespread use in private sector auctions.\(^1\)

The cornerstone result of auction theory, the Revenue Equivalence Theorem, tells us as a corollary that optimal reserve prices are identical among a broad class of auction schemes.\(^2\) Myerson (1981, 1983) demonstrated that a discriminatory reserve policy is an important component of an optimal auction when bidders are distributionally heterogeneous. Graham, Marshall, and Richard (1987, 1989) showed that an English auctioneer who is facing heterogeneous bidders will use a reserve price that is a function of the bid sequence, thus providing an explanation for the phenomena of phantom bidding. Finally, Graham and Marshall (1987) and Mailath and Zemsky (1989) have demonstrated that an auctioneer who is facing a bidder coalition at either a second price or English auction will invoke a reserve price that is an increasing function of the size of the coalition.

The federal government often uses reserve prices in the sale of commodities. For example, when selling timber from federal land a specific appraisal system is used to determine a reserve price which is announced prior to the submission of bids. When selling offshore oil the federal government sets a pre-announced reserve price on each tract. However, these reserve prices do not seem to reflect the government's monopoly power. In other words, the reserves do not appear to be optimally set.

The information regarding the use of a reserve price in federal procurements is mixed. To the best of our knowledge a reserve price has never been explicitly stated in an Invitation for Bid (IFB) or a Request for Proposals (RFP). Nevertheless, both the Federal Acquisition Regulations (FAR) and the Defense Acquisition Regulations (DAR) specifically allow for the use of a reserve price and, naturally, allow a procurement official to make no award. In fact, "non-awards" are observed, albeit infrequently, in both defense and non-defense procurements.

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\(^1\) Cassady (1967) provides an excellent discussion of the use of reserves at auctions for a wide variety of commodities.

\(^2\) As in Riley and Samuelson (1981).
Two points are important to note regarding "no-awards". First, officials involved with overseeing the federal procurement process have informed us that "no-awards" usually stem from constraints that are not related to reserve prices, such as the small business set-aside program, the Buy American Act, and insufficient budgetary authorization.

Second, although the FAR and DAR refer to the construction of "should costs" and other types of price estimates that are to be constructed by procurement officials there is no explicit reference to the calculation of an optimal reserve price.\(^3\) It appears that the primary goal of the price estimates is to prevent excessively low bids (low-ball bids) from winning procurements.\(^4\)

The best evidence regarding the lack of use of optimal reserves in federal procurements comes from Office of Management and Budget (OMB) Circular A–76. In essence, A–76 strongly recommends the replacement of "in-house" government production with competitively obtained private sector supplies. Consider the following four passages from A–76 (revised 8/4/83).

"In the process of governing, the Government should not compete with its citizens. The competitive enterprise system, characterized by individual freedom and initiative, is the primary source of national economic strength. In recognition of this principle, it has been and continues to be the general policy of the Government to rely on commercial sources to supply the products and services the Government needs."

"Whenever commercial sector performance of a Government operated activity is permissible, ..., comparison of the cost of contracting and the cost of in-house performance shall be performed to determine who will do the work."

\(^3\)When using the word "optimal" we are implicitly assuming that the government's objective is to maximize its expected surplus. It might be reasonable to presume that the government is also concerned with the expected surplus of suppliers as well. However, this reasoning is not consistent with explicit directives to procurement personnel in the FAR.

"The contracting officer's primary concern is the price the Government actually pays; the contractor's eventual cost and profit or fee should be a secondary concern." (FAR 15.803–d)

\(^4\)The relevant sections of the FAR are 14.404–1–c6, 15.610–d3, 15.802–d, 15.805–2, and 15.810.
"... the Government shall not start or carry on any activity to provide a commercial product or service if the product or service can be procured more economically from a commercial source."

"Government performance of a commercial activity is authorized if a cost comparison ... demonstrates that the Government is operating or can operate the activity on an ongoing basis at an estimated lower cost than a qualified commercial source."

In our opinion, the abandonment of government production constitutes, for many procurements, the elimination of the possibility of using an optimal reserve price. In the absence of in-house production the use of an optimal reserve price implies that the government will occasionally not procure at all. For many procurements the threat of no-award is only credible if the government can produce the item itself. Since A-76 does not account for the loss of bargaining power that results from the abandonment of in-house production it seems reasonable to conclude, in conjunction with other evidence, that optimal reserve prices are not utilized in federal procurements.

In this paper we analyze the impact of conducting procurements with and without the use of optimally determined reserves. An obvious and immediate result from the literature is that the government obtains less surplus without an optimal reserve than with one. The intuition for this result is simple. A procurement official who can employ a reserve policy can choose to use one that is non-binding for all types of bidders. Consequently, in the absence of an agency problem, the surplus obtained by a procurement official who can select a reserve will never be lower than the surplus obtained by a procurement official who cannot employ a reserve price.

More interestingly, under reasonable conditions the government procures lower levels of technology with no reserve price policy than when an optimal reserve price is

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5If the procuring agency has a "special" relationship with a particular supplier then this contractor can be a substitute for in-house production and, consequently, permit the agency to credibly employ an optimal reserve. Further analysis of this mutually beneficial and frequently observed relationship can be found in our paper entitled, "The Credibility of an Auctioneer's Reserve through the Use of Protecting Bidders" (Marshall, Meurer, and Richard – 1990).

6To the best of our knowledge, federal procurement regulations do not enable an agency to commit to a policy of not re-auctioning an item that was not procured.
employed. The intuition for this result is as follows. The procurement official must simultaneously select a technology level for the procurement and, if applicable, a reserve price. The optimal technology choice involves a trade-off between the marginal utility gain from a higher technology and the marginal cost increase. The marginal cost increases both directly and indirectly — the former occurs because quality improvements imply higher marginal costs while the latter stems from the fact that fewer firms can meet the stricter specifications. Relative to a case where a reserve price is not employed, a reserve price may reduce the expected marginal procurement cost for a given technology level. Consequently, with a reserve price both the technology level and expected surplus will be higher than that of an unreserved procurement. The reserve price effects the procurement in a manner not unlike the addition of another bidder.

In addition, we allow for the possibility that the procurement official suffers from a particular form of agency problem that we call a "technology bias". The technology bias implies that, relative to taxpayers, the procurement official uses an objective function that places disproportionately heavy emphasis on the quality/technology of the product and relatively little weight on cost. With this objective function we demonstrate that the use of an optimal (from the viewpoint of the procurement official) reserve may permit the procurement official to acquire a high level of technology at extreme cost — not "extreme" to the procurement official but extreme to taxpayers. We provide a characterization of cases where the surplus to taxpayers is higher when a reserve is NOT used than when its use is permitted. Intuitively, the agency problem will distort the marginal conditions for determination of an optimal technology level in favor of higher technology items. The use of a reserve can, in the absence of an agency problem, result in selection of a higher technology level. The agency problem will exaggerate this effect further through the down-weighting of costs and up-weighting of technology. Consequently, in the presence of a technology bias, taxpayers might be better off banning the use of a reserve price — by doing so they achieve a procurement that is closer to their objective of surplus maximization. This phenomena provides a potential explanation as to why the FAR and DAR have developed to a point where reserves are all but explicitly forbidden.

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7We model the choice of technology as being a minimum specification where no advantage accrues to a bidder that bids a technology level above the minimum specification (assuming that higher technology products cost more to produce). For many procurements it is relatively difficult to measure quality differentials between products but relatively simple to determine if a product meets the minimum specifications. Consequently, the lowest cost bidder wins in many government procurements (known as IFB's).
Finally, suppose that the government can adopt a mechanism for monitoring the actions of procurement officials that eliminates or severely dampens the agency problem discussed above. Then the use of reserve policy will not only result in higher expected surplus for taxpayers but will permit the procurement official, who suffers from an agency problem but cannot exhibit it due to the monitoring mechanism, to procure higher levels of technology than would be possible without a reserve policy. In other words, with a sufficiently strong procurement oversight mechanism in place, both taxpayers and technology biased procurement officials will be better off with a reserve policy.

The paper proceeds as follows. In Section I we discuss some simple examples to illustrate a number of the critical issues and results. The models and formal results are presented in Section II. Concluding remarks are offered in Section III.

I. Examples

We present two examples. The first one illustrates known results or immediate implications of known results regarding reserve pricing at procurements. The second illustrates the main results of our paper.

Example 1:

The government wants to obtain one unit of a standardized item by means of a second-price sealed bid procurement. The government will derive benefits of $\bar{U}$ from the item. Its objective is to maximize the expected surplus from the procurement which, in this case, is equivalent to minimization of the expected cost. $N$ risk neutral bidders will participate in the procurement. They independently and privately draw their costs ($V_i$) of production from the uniform distribution with lower support of zero and upper support of one ($V_i \sim U(0,1)$, $\forall i=1,\ldots,N$). Initially, we assume that the government does not possess an in-house production capability. For all firms and the government we assume that fixed costs are zero and the marginal costs are constant. If the procurement is conducted without a reserve then the expected cost of procurement is $2/(N+1)$, the unconditional expectation of the second lowest cost. If a reserve can be employed then the optimal reserve is
$S^* = \overline{U}/2$. Expected utility is reduced by $\overline{U}(1-S^*)^N$, where $(1-S^*)^N$ represents the probability of no-award. Expected procurement costs are reduced even further and the government expected surplus can be written as

$$GS^* = \left[\overline{U} - \frac{2}{N+1}\right] + \frac{2}{N+1} (1-S^*)^{N+1}$$

(1)

Other reserve levels could be used for a variety of reasons (one discussed below) but they would not be optimal. The government's expected surplus resulting from the use of a suboptimal reserve level $S \in (0,1)$ is given by

$$GS = \left[\overline{U} - \frac{2}{N+1}\right] + \frac{2}{N+1} (1-S)^N \left[\frac{1}{(N+1)} \frac{(1-S^*)^N}{(1-S)} - N(1-S)\right]$$

(2)

GS is clearly less than GS*. Nevertheless, GS exceeds the expected surplus in the absence of a reserve as long as $N \cdot S > (N+1) \cdot S^* - 1$.

As discussed in the introduction, the possibility of no-award raises a credibility issue. Along these lines, in-house production can be incorporated in the above example.

Suppose the government possesses the capability of producing the item in-house for a marginal cost of $c_g$ where $\overline{U} \geq c_g > 0$. Then the optimal reserve is $Z^* = c_g/2$ and the expected surplus is the same as above with $Z^*$ replacing $S^*$.

It is important to understand why $c_g$ itself is not the optimal reserve. Clearly, there is no reason for the government to pay in excess of its own marginal cost of production. There are two consequences of setting a reserve $Z$ below $c_g$.

(1) If the lowest cost $V_1$ is less than $Z$ while the second lowest $V_2$ is higher than $Z$, then the government pays $Z$. With a reserve of $c_g$ the government would instead pay $\text{Min}(V_2, c_g)$.

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In order to focus our attention on the main issue of interest it is assumed that $2 \geq \overline{U} > 2/(N+1)$.
(ii) If \( V_1 \) and \( V_2 \) are both in the interval \( (Z, c_g) \) then the government "pays" \( c_g \). With a reserve of \( c_g \), the government would instead pay \( V_2 \).

When \( Z \) is close enough to \( c_g \), the first effect dominates the second one. Hence, the optimal reserve \( Z^\star \) is lower than \( c_g \). In fact, \( Z^\star = c_g/2 \) in our example. Here again other reserve levels could be used that would not be optimal. Note, finally, the strategic use of in-house capacity: when \( Z^\star < V_1 < c_g \), the government relies upon in-house capacity despite the availability of a lower cost producer. In compensation for this apparent inefficiency, an in-house production capability prevents the government from not procuring at all.

**Example 2.**

In this example we assume that the government must select both a technology level, \( x \), and a reserve price, \( S \), to maximize expected surplus. The cost function faced by firm \( i \) is given by \( C_i(V_i, x) \) where \( V_i \) is the firm's "cost index". The firms independently and privately draw their random costs indices \( (V_i) \) from the uniform distribution with lower support of zero and upper support of one \( (V_i \sim U(0, 1)) \forall i = 1, ..., N \). The government possesses no in-house production capability. The utility derived from obtaining an item of technology level \( x \) is \( U(x) \). Firms derive no competitive advantage from supplying a technology level above \( x \) once the government has announced, via an IFB, \( x \). In other words, \( U(x) \) is not contingent on bids submitted by potential vendors.

As in Example 1, the imposition of an optimal reserve increases the government's expected surplus. It also affects both the marginal utility and marginal cost of procuring a given technology level. Whether or not the imposition of a reserve increases the optimal technology level depends on how these two marginal quantities vary relative to each other.

A trivial example is where \( C_i(V_i, x) \) is additively separable in \( V_i \) and \( \phi(x) \), a non-random cost with \( \phi'(x) > 0 \).

\[
C_i(V_i, x) = V_i + \phi(x)
\]

Marginal utility and cost are then affected in exactly the same way by the imposition of a reserve since \( \phi(x) \) is simply subtracted from \( U(x) \). It follows that the imposition of a reserve does not affect the choice of an optimal technology level.
A more interesting example is where $C_i(V_i,x)$ is multiplicatively separable in $V_i$ and $\phi(x)$.

\begin{equation}
C_i(V_i,x) = V_i \cdot \phi(x)
\end{equation}

Note that imposing a reserve $R$ on costs is equivalent to setting an upper bound $S=R/\phi(x)$ on the cost index $V_i$. (The latter is more convenient analytically.) In order to avoid ambiguities we refer to $R$ and $S$ as the reserve and the "reserve index", respectively.

In the absence of a reserve, the government selects a technology level $x_1^*$ that maximizes expected surplus.

\[ GS_1 = U(x) - \frac{2}{N+1} \cdot \phi(x) \]

Let $GS_1^* = GS_1(x_1^*)$. When a reserve is available the optimal reserve index and the corresponding expected surplus are given by

\begin{equation}
S^*(x) = \frac{1}{2} \cdot \frac{U(x)}{\phi(x)}
\end{equation}

\begin{equation}
GS_2(x) = GS_1(x) + \frac{2\phi(x)}{N+1} \cdot \left[ 1 - S^*(x) \right]^{N+1}
\end{equation}

Let $x_2^*$ denote that technology level that maximizes $GS_2(x)$ and let $GS_2^* = GS_2(x_2^*)$. Obviously, $GS_2^* \geq GS_1^*$. The relation between $x_1^*$ and $x_2^*$ is less obvious and explicitly addressed below.

As discussed in the introduction, procurement officials may suffer from a "technology bias" whereby costs are downweighted relative to the utility of the procured technology. Under our interpretation of the "technology bias" the procurement official's objective function is the same as that of the government, except that all cost factors are multiplied by a factor $\lambda \in (0,1)$. Hence, for a given technology level $x$, the procurement official selects a reserve index.
(8) \[ S^\lambda(x) = \frac{1}{\lambda} S^*(x) \]

Furthermore, with costs being downweighted the procurement official selects a technology level \( x_2^\lambda \) which is typically (much) larger than \( x_2^* \). The procurement official's expected surplus is inconsequential. The essential issue is the government's expected surplus when reserves and technologies are chosen by the procurement official. Given \( x \), the choice of \( S^\lambda(x) \) instead of the socially optimal \( S^*(x) \) generates the following expected surplus for the government.

\[ GS^\lambda_2(x) = GS^*_1(x) + \frac{2\phi(x)}{N+1} \left[ 1 - \frac{S^*(x)}{\lambda} \right]^N \]

(9)

\[ \cdot \left[ (N+1)(1 - S^*(x)) - N \left[ 1 - \frac{S^*(x)}{\lambda} \right] \right] \]

or, under the technology \( x_2^\lambda \)

(10) \[ GS^\lambda = GS^\lambda_2 \left[ x_2^\lambda \right] \]

Each of the two factors in (9) could be negative as the result of the technology biased choice of the procurement official. In the example below it turns out that \( S^\lambda_2(x_2^\lambda) = S^*_2(x_2^*) = S^*_2 \) so that the procurement official's decision affects the government's expected surplus solely through the factor \( GS^*_1(x_2^\lambda) \).

Consider the following special case.

(11) \[ U(x) = x^\beta, \phi(x) = x^\alpha, \text{ and } N = 2 \]

with \( 3\beta > \alpha > 2\beta > 0 \) (these restrictions suffice for the second order conditions and for \( 0 < S^*_2 < 1 \)). A complete analytical solution exists for this case which is given by
(12) \[ x_1^* = \left[ \frac{1}{2\alpha} \right]^{\frac{1}{\alpha - \beta}} \beta \] \quad GS_1^* = \left[ x_1^* \right]^\beta \left[ 1 - \frac{\beta}{\alpha} \right]

(13) \[ S_2^* = S_2 = 3 \cdot \frac{\alpha - 2\beta}{2\alpha - 3\beta} \]

(14) \[ x_2^* = \left( \frac{1}{2S_2^*} \right)^{\frac{1}{\alpha - \beta}} \beta \] \quad GS_2^* = \left[ x_2^* \right]^\beta \cdot S_2^* \cdot \left[ 2 - S_2^* \right] \cdot \left[ 1 - \frac{\beta}{\alpha} \right]

(15) \[ x_1^* = \left( \frac{\lambda - 1}{\alpha - \beta} \right) \beta \] \quad GS_1^* = \left[ \frac{1}{\alpha - \beta} \right]^\beta \cdot \left[ \frac{\alpha \lambda - \beta}{\alpha - \beta} \right] \cdot GS_1^*

Note, in particular, that

\[ GS_2^* > GS_1^* \] and \[ x_2^* > x_1^* \]

This example suggests that any federal regulations which inhibits the use of an optimal reserve policy by procurement officials is not only objectionable on surplus grounds but, in addition, can result in the procurement of lesser or inferior technology. If \( \lambda > \beta / \alpha \), then we also have \( GS_2^* > GS_1^* \), so that the imposition of a reserve remains favorable to the government despite technology biased choices. If, however, \( \lambda < \beta / \alpha \) then the \( GS_1^* \)'s are negative and \( GS_2^* < GS_1^* \). In other words, in the presence of the specific agency problem posed here the government attains higher expected surplus (albeit negative in both cases) from the actions of a procurement official who is constrained NOT to employ a reserve policy than from one who can use an optimal reserve. Specific numerical values for the above example are presented in Table 1.
Table 1

\[ \alpha = 1.2, \beta = 0.5, N = 2 \]

<table>
<thead>
<tr>
<th></th>
<th>( \lambda = 1 )</th>
<th>( \lambda = 0.7 )</th>
<th>( \lambda = 0.4 )</th>
<th>( \lambda = 0.1 )</th>
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<tr>
<td>( S_2^* )</td>
<td>0.667</td>
<td>0.667</td>
<td>0.667</td>
<td>0.667</td>
</tr>
<tr>
<td>Pr(no award)</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
</tr>
<tr>
<td>( x_1^* \lambda )</td>
<td>0.511</td>
<td>0.851</td>
<td>1.892</td>
<td>13.708</td>
</tr>
<tr>
<td>( x_2^* \lambda )</td>
<td>0.663</td>
<td>1.104</td>
<td>2.455</td>
<td>17.786</td>
</tr>
<tr>
<td>GS_1^* \lambda \ )</td>
<td>0.417</td>
<td>0.373</td>
<td>-0.057</td>
<td>-11.724</td>
</tr>
<tr>
<td>GS_2^* \lambda \ )</td>
<td>0.422</td>
<td>0.378</td>
<td>-0.058</td>
<td>-11.871</td>
</tr>
</tbody>
</table>

II. Models and General Results

The government is to procure a single object from one of \( N \) potential vendors. The government must announce a minimally acceptable technology or quality level, \( x \), for the product prior to the submission of bids. Any bid describing a product below \( x \) is automatically rejected. Any bid describing a product in excess of \( x \) is valid and will be evaluated solely on cost criteria.\(^9\) The government derives \( U(x) \) from the acquisition of technology level \( x \) where \( U'(x)>0 \).

Firms have ex ante random costs of production. In particular, \( C_i = \phi(V_i, x) \) where \( V_i \) is a cost index that is uniformly distributed on \((0,1)\).\(^{10}\) In addition, we assume that \( \phi_V > 0 \),

\(^9\)We are providing a stylized description of an IFB procurement. In its extreme variant, the government decides on \( x \) and then, regardless of the quality or technology of the products offered by valid bidders, evaluates all of them as if they were of level \( x \). We adopt this extreme variant so that we can discuss a reserve price in the context of cost rather than within the context of surplus.

\(^{10}\)This formulation characterizes cost distributions where cost is "affiliated" with technology. Let \( F(c|x) \) denote the distribution function of cost given technology. If its density function is strictly positive and if

\[ \frac{\partial}{\partial x} F(c|x) < 0, \]
\( \phi_x > 0 \), and, therefore, \( \phi(V,x) \) is invertible for a given \( x \). Under these assumptions on \( \phi \), setting a reserve \( R \) on cost is equivalent, given technology level \( x \), to setting a reserve index \( S \) on the cost parameter \( V \). \( (\phi(V,x) < R \Leftrightarrow V < S \text{ where } R = \phi(S,x)) \).

We restrict attention to mechanisms where the lowest cost firm will be named as awardee (if its bid is lower than the reserve). Further, no firm is favored ex ante by the procurement rules. We do not analyze reserve pricing for a specific procurement mechanism but, rather, following Riley and Samuelson (1981), we pose the competitive procurement with sufficient generality that any mechanism which satisfies our assumptions is covered as a special case.

Throughout the main text \( N \) is a random variable whose distribution is assumed to be independent of \( x \). Let \( p_n = \Pr(N = n), \ n \geq 0 \). In the Appendix we demonstrate how the results of Section II.A would change if the distribution of \( N \) "shifts to the left" as \( x \) increases.

II.A The Relation Between Optimal Reserves and Procured Technology

Given a choice of \( x \) by the government we denote the expected payment to a firm that privately reports a cost index \( b \) as \( P(b,x) \). The expected profit of a firm with cost index \( v \) if it chooses to enter the procurement is then:

\[
\Pi(x,v,b) = P(b,x) - \phi(v,x) \cdot L(b)
\]

where \( L(b) \) is the (marginal) probability that \( b \) is the lowest report

\[
L(b) = \sum_{n=0}^{\infty} p_n \cdot (1-b)^{n-1}
\]

For reservation cost \( S \) we have

\[
\Pi(x,S,S) = 0
\]

then the random variable \( V = F(C|x) \) is uniform on \([0,1]\) and \( \phi \) is obtained by inverting \( F \), given \( x \).
In addition,

\[
\frac{\partial}{\partial b} \Pi(x, v, b) \bigg|_{b=v} = 0 \quad v \leq S
\]

which, after substitutions and integration by parts, implies

\[
P(v, x) = \phi(v, x)L(v) + \int_{v}^{S} \phi_u(u, x) \cdot L(u) \, du
\]

The government's expected payment to a particular firm is then

\[
\bar{P}(x) = \int_{0}^{S} P(v, x) \, dv
\]

\[
= \int_{0}^{S} [\phi(v, x) + v \phi_v(v, x)] \cdot L(v) \, dv
\]

\[
= \int_{0}^{S} h(v, x) \cdot L(v) \, dv
\]

Therefore, the expected cost to the government is \(N \cdot \bar{P}(x)\).

It is important to note that the above derivation and expression for the expected payment to the government permits us to analyze the problem without reference to a particular procurement mechanism, such as first price or second price.

Using expression (21) we can now write the objective function for the government, whether a reserve is being employed or not. Let

\[
Q(v) = \Pr(N \geq 1, V_1 \geq v) = \sum_{n=1}^{\infty} p_n (1 - v)^n
\]
(23) \[ q(v) = -\frac{d}{dv} Q(v) = \sum_{n=1}^{\infty} n \cdot p_n \cdot (1-v)^{n-1} \]

and note that

(24) \[ Q(0) = 1 - p_0, \quad q(0) = E(N), \quad Q(1) = q(1) = 0 \]

For the case where no reserve is employed we have

(25) \[ \max_x \left[ U(x) \cdot Q(0) - \int_0^1 h(v,x) \cdot q(v) dv \right] \]

whereas, when a reserve of S is employed we have the following objective function

(26) \[ \max_{x,S} \left[ U(x) \cdot [Q(0)-Q(S)] - \int_0^S h(v,x) \cdot q(v) dv \right] \]

The first order condition for (25) provides an implicit solution for the optimal technology level in the absence of a reserve.

(27) \[ U'(x_1^*) = \Psi(0,x_1^*) \]

where

(28) \[ \Psi(a,x) = \frac{1}{Q(a)} \int_a^1 \frac{\partial}{\partial x} h(v,x) \cdot q(v) dv \]

The first order conditions for (26) provides an implicit solution for the optimal reserve price and the corresponding optimal technology level.

(29) \[ U(x_2^*) = h(S_2^*,x_2^*) \]

and
\[(30) \quad \left[ U'(x_2^*) - \Psi(0,x_2^*) \right] \cdot Q(0) = \left[ U'(x_2^*) - \Psi(S_2^*,x_2^*) \right] \cdot Q(S_2^*) \]

Furthermore, let $S_1^*$ denote the best reserve to use when technology level $x_1^*$ is chosen. In other words,

\[(31) \quad U(x_1^*) = h(S_1^*,x_1^*) \]

The first order conditions above lead to the following theorem (assuming second order conditions are satisfied).

**Theorem 1:**

If the objective function (26) is quasi-concave, then a necessary and sufficient condition for $x_2^* > x_1^*$ is

\[(32) \quad \Psi(S_1^*,x_1^*) > \Psi(0,x_1^*) \]

**Proof:**

Evaluating the partial derivative of (26) with respect to $x$ at $S_1^*,x_1^*$ yields the expression below.

\[(33) \quad Q(0) \cdot \left[ U'(x_1^*) - \Psi(0,x_1^*) \right] - Q(S_1^*) \cdot \left[ U'(x_1^*) - \Psi(S_1^*,x_1^*) \right] \]

By (27) the first term in parentheses in (33) is zero. Given (32) the second expression in parentheses is negative which implies that (33) is positive and, by quasi-concavity of the objective function, $x_2^* > x_1^*$. This establishes sufficiency. If $x_2^* > x_1^*$ then (33) is positive. The first term in parentheses is still zero by (27). Necessity follows by replacing $U'(x_1^*)$ with $\Psi(0,x_1^*)$ in the second parenthetical expression. QED.
Condition (32) is more easily interpreted when it is rewritten using the following definition.

\begin{equation}
\text{MEU}(S,x) = \frac{\partial}{\partial x} \left[ U(x) \cdot [Q(0) - Q(S)] \right]
\end{equation}

\begin{equation}
\text{MEC}(S,x) = \frac{\partial}{\partial x} \int_0^S h(v,x) \cdot q(v) dv
\end{equation}

For a given reserve \( S \), \( \text{MEU}(S,x) \) and \( \text{MEC}(S,x) \) are the marginal expected utility and cost of procuring a technology level \( x \). Condition (32) can now be rewritten as

\begin{equation}
\frac{\text{MEU}(S_1^*,x_1^*)}{\text{MEU}(1,x_1^*)} > \frac{\text{MEC}(S_1^*,x_1^*)}{\text{MEC}(1,x_1^*)}
\end{equation}

In words, when a reserve of \( S_1^* \) is used the additional utility obtained from \( x_1^* \) per dollar spent to acquire it must exceed the additional utility obtained from \( x_1^* \) per dollar spent to acquire it when no reserve is used. Or, the marginal utility per dollar spent to obtain \( x_1^* \) is bigger with a corresponding optimal reserve than with no reserve. Given that \( x_1^* \) is optimal with no reserve, by standard marginal analysis it must be the case from (36) that the optimal choice of technology when a reserve is employed exceeds \( x_1^* \).

If we assume that the cost function is separable in \( x \) and \( V \) then additional results are possible. Specifically, let

\[ \phi(v,x) = \phi_1(v) \cdot \phi_2(x), \quad h(v,x) = h_1(v) \cdot \phi_2(x), \quad \text{with} \]

\begin{equation}
\phi_1(v) = \phi_1(v) + v \cdot \phi_1'(v)
\end{equation}

Then we have the following theorem.
Theorem 2:

Under separability as given in (37),

(i) the condition

\[
\int_{S_1^*}^1 h_1(v) \cdot q(v) \, dv > \frac{U'(x_1^*) \cdot \phi_2(x_1^*)}{h_1(S_1^*) \cdot Q(S_1^*)}
\]

is necessary and sufficient for \( x_2^* > x_1^* \), and

(ii) the conditions

\[
h_1'(v) \text{ for } v > S_1^*, \text{ and}
\]

\[
\frac{\phi_2'(x_1^*)}{\phi_2(x_1^*)} > \frac{U'(x_1^*)}{U(x_1^*)}
\]

are sufficient for \( x_2^* > x_1^* \).

Proof:

By (27), (28), and (32) we find that the condition

\[
\frac{\phi_2'(x_1^*)}{Q(S_1^*)} \int_{S_1^*}^1 h_1(v) \cdot q(v) \, dv > U'(x_1^*)
\]

is necessary and sufficient for \( x_2^* > x_1^* \). Division by \( h_1(S_1^*) \cdot \phi_2'(x_1^*) \) together with (31) completes the proof of (38). (39) follows from the fact that if \( h_1'(v) > 0 \) for \( v > S_1^* \), then the left-hand term in (38) is greater than 1. QED.
Of most interest from Theorem 2 is the sufficient condition (40). In order to
determine if an optimal reserve policy will lead to a higher level of technology it is not
necessary for an external policy evaluator to know anything about the distributional source
of private cost information or even the number of bidders who could potentially participate
in the procurement. All the policy maker must be able to evaluate is whether the
technology-specific cost of procurement is rising faster in percentage terms than the utility
derived from the technology procured. It is only if that condition fails to hold that the
more complex condition (38) needs to be examined.

It seems extremely plausible for DoD weapon procurements that technology-specific
costs are increasing faster than the utility derived from their procurement. Theorem 2 tells
us that in such procurements the DoD would be able to secure even higher levels of
technology if an optimal reserve policy were employed. If "our boys in uniform deserve the
best possible equipment"\textsuperscript{11} then the DoD should invoke an optimal reserve policy on all
weapon procurements.

II.B The Impact of a Technology Bias on Taxpayer Surplus

The above analysis relies on the assumption that the procurement official handling a
federal procurement is acting to maximize expected surplus from the procurement.
However, there is substantial evidence that procurement officials, especially in the DoD,
suffer from a technology bias. Specifically, relative to surplus maximizing behavior, they
place far greater emphasis on the technology of the procured item than the cost. Consider
the following passages from The Defense Game by Richard Stubbing.

"... modern fighters are typically equipped, at great expense, with the capability to
fly at speeds of Mach 2 or better, yet this capability is of little use in combat."
(page 155)

"Similar arguments are heard in respect to the advantages of nuclear powered versus
diesel–electric submarines which cost one–third as much, computer guided 'smart'
missiles compared to much cheaper but historically effective heat–seeking missiles,
computer aimed versus hand–aimed anti–aircraft guns. In most cases, the added
capability is 'nice to have', but the key question, too often ignored, is whether it is

\textsuperscript{11}From Stubbing (page 153), who was paraphrasing President Reagan.
necessary when a marginally less effective system can be bought at a far lower price." (page 155)

Given the existence of a technology bias, is it possible that taxpayers are better off when procurement officials are not permitted to employ an optimal reserve policy? In other words, could the use of an optimal reserve lead to the procurement of such a high level of technology that taxpayer surplus would be higher when the lower technology associated with no reserve is procured? Example 2 in Section I demonstrates that this is indeed possible. A more general statement of the result is presented below.

As discussed previously, the technology bias is represented via a single parameter $\lambda$. The specification we now consider is characterized by

$$U(x) = x^\beta ; \quad \phi(v,x) = \phi_1(v) \cdot x^\alpha$$

In contrast with Example 2, both $\phi_1(v)$ and the distribution of $N$ are left arbitrary. Let

$$\delta(a) = \int_0^a h_1(v) \cdot q(v) dv$$

with $h_1(v) = \phi_1(v) + v \cdot \phi_1'(v)$

The objective functions of the procurement official for the no-reserve and reserve cases are given by formulae (25) and (26), respectively, except that the costs are downweighted by the parameter $\lambda$. It is implicitly assumed that $\alpha$ and $\beta$ satisfy the appropriate conditions required for the existence of interior solutions.

In the absence of a reserve the government's optimal technology level $x_1^*$ and surplus $GS_1^*$ are derived from condition (27) and may be written as

$$x_1^* = \left[ \frac{\alpha \cdot Q(0)}{\beta \cdot \delta(1)} \right]^{1/\alpha - \beta}$$
\[
GS_1^* = \left[ x_1^* \right]^\beta \cdot Q(0) \cdot \left[ 1 - \frac{\beta}{\alpha} \right]
\]

A similar derivation with \( \lambda \) multiplying \( h(v,x) \) in (25) and (27) yields the procurement official's optimal technology level \( x_1^{*\lambda} \) and the corresponding surplus to the government of \( GS_1^{*\lambda} \). It is easily shown that the relationship between the two sets of solutions is still given by (15) for \( i=1 \).

The solution for the reserve case is derived from conditions (29) and (30) — with and without the parameter \( \lambda \) multiplying cost — and we find that

\[
\alpha \cdot \delta \left[ S_2^* \right] = \beta \cdot h_1 \left[ S_2^* \right] \cdot \left[ Q(0) - Q \left[ S_2^* \right] \right]
\]

\[
\left[ x_2^* \right]^\beta - \alpha = \frac{1}{h_1 \left( S_2^* \right)}
\]

\[
GS_2^* = \left[ x_2^* \right]^\beta \left[ Q(0) - Q \left[ S_2^* \right] \right] \cdot \left[ 1 - \frac{\beta}{\alpha} \right]
\]

We also find that \( S_2^{*\lambda} = S_2^* \) and that condition (15) still applies for \( i=2 \). If, in particular, \( \alpha \lambda < \beta \) then \( GS_1^{*\lambda} > GS_2^{*\lambda} \) even though \( GS_2^* > GS_1^* \) and the government is better off without a reserve price (though the problem is less that of a suboptimal reserve choice than that of the choice of an extraordinarily high technology level).

III. Conclusion

The results in this paper appear to have a mixed message. First, it is shown that optimal reserve prices have an impact on procurements beyond the gains in expected surplus. Specifically, under reasonable conditions, higher technology levels are attained when optimal reserve prices are employed than when no reserve policy is used. This result, viewed in isolation, implies the need for deregulation of the procurement process. In particular, relevant sections of the DAR, FAR, and OMB A–76 should be changed or removed to encourage the use of an optimal reserve by procurement officials.
However, if a given procurement official suffers from a specific agency problem which we refer to as a technology bias then the use of an optimal reserve might lead to the procurement of such a high level of technology that taxpayers would be better off, in terms of expected surplus maximization, if the procurement official was banned from using a reserve policy.

It is likely that federal procurement regulations with respect to reserve pricing have reached their current status for good reasons. If a technology bias or some equivalent type of agency problem has been pervasive among procurement officials then it is reasonable to expect that representatives of taxpayers have reacted to it by limiting the extent to which the agency problem results in acquisitions that are different from expected surplus maximization.

It is important to note that we are not advocating an ever increasing expansion of the FAR and DAR in order to curb agency problems among procurement officials. Quite to the contrary, in our opinion these individuals should be given greater discretion in acquisition decisions than current regulations allow. However, given the existence of pervasive agency problems it is important to oversee the procurement process in order to deter and correct these problems. Fortunately, a very promising development has occurred on this front in recent years, at least with respect to federal procurements of computers and telecommunication equipment. Excluded and/or losing bidders in these procurements have the right to protest any aspect of the procurement process to a quasi-judicial board. We have recently demonstrated (Marshall, Meurer, and Richard – 1989a, 1989b) that this new protest process has been extremely successful in deterring procurement officials from exhibiting a technology bias in their acquisitions. It has had numerous other socially beneficial effects as well, such as encouraging entry into a market where newcomers were not previously welcome.

In our opinion, an effective oversight process can make much of the FAR and DAR unnecessary. In particular, the protest process might function so well in curbing agency problems that taxpayers will definitively benefit from the use of optimal reserve prices by procurement officials. If this happens then procurement officials will be able to at least partially quench their desire for "high-tech" items through the use of reserve prices.
BIBLIOGRAPHY


APPENDIX

The purpose of the Appendix is to identify the changes that would be required in Section II.A in order to sustain analogous results when the distribution of N depends on x. This case is of interest since it is reasonable that the number of firms able to provide a very high level of technology (say, an 80486 PC) is smaller than the number who can provide a lower level of technology (say, an 80286 PC).

The major substantive change is to replace $\Psi$ with $\bar{\Psi}$ in (29) and subsequent formulas.

\[
\bar{\Psi}(a,x) = \frac{1}{Q(a,x)} \int_a^1 \left\{ \frac{\partial}{\partial x} h(v,x) + \left[ h(v,x) - h(S^*(x),x) \right] \right\} \cdot q(v,x) dv
\]

(48)

where

\[
Q(a,x) = \sum_{n=1}^{\infty} p_n(x) \cdot (1-a)^n ; \quad p_n(x) = \Pr(N=n|x) ; \quad q(a,x) = -\frac{\partial}{\partial a} Q(a,x)
\]

(49)

Then, Theorem 1 and its proof are nearly identical to those in the main text with $\bar{\Psi}$ replacing $\Psi$.

We can also generalize Theorem 2 and, in particular, find conditions under which the sufficient condition (40) still applies. We need an additional component of the separability conditions in (37). We assume that

\[
p_0(x) = 1 - (1-p_0) \cdot q_2(x)
\]

(50)

\[
p_n(x) = p_n \cdot q_2(x)
\]

with
(51) \[ q_2(0) = 1, \quad q'_2(x) < 0, \quad \lim_{x \to \infty} q_2(x) = 0, \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = 1. \]

It follows that

(52) \[ Q(v, x) = Q(v) \cdot q_2(x) \quad \text{and} \quad q(v, x) = q(v) \cdot q_2(x) \]

where \( Q(v) \) and \( q(v) \) are defined in (22) and (23), respectively.

**Theorem 2:**

Under separability as given in (37) and (52)

(i) the condition

(53) \[ \left[ \frac{\phi_2'(x^*_1)}{\phi_2(x_1)} + q'_2(x_1) \right] \cdot \frac{\int_{S_1^*}^{1} h_1(v) \cdot q(v) dv}{h_1(S_1^*) \cdot Q(S_1^*)} > \frac{U'(x_1^*)}{U(x_1)} + \frac{q'_2(x_1^*)}{q_2(x_1)} \]

is necessary and sufficient for \( x_2^* > x_1^* \), while

(ii) the conditions

(54) \[ \frac{d}{dx} \left[ \phi_2(x) \cdot q_2(x) \right] \bigg|_{x=x_1^*} > 0 \]

(39) \[ h_1'(v) > 0 \quad \text{for} \quad v \geq S_1^*, \quad \text{and} \]

(40) \[ \frac{\phi_2'(x_1^*)}{\phi_2(x_1^*)} \quad \text{and} \quad \frac{U'(x_1^*)}{U(x_1^*)} \]

are sufficient for \( x_2^* > x_1^* \).
Proof:

The proof is similar to that of Theorem 2, except that $\Psi$ is replaced by $\tilde{\Psi}$. By (27), (48) and (32) we find that

$$
\frac{\phi_2(x_1^*) \cdot q_2(x_1^*) + \phi_2(x_1^*) \cdot q_2(x_1^*)}{Q_1(S_1^*) \cdot q_2(x_1^*)} \cdot \int_{S_1^*}^{1} h_1(v) \cdot q(v) dv
$$

$$
> U'(x_1^*) + h_1(S_1^*) \cdot \phi_2(x_1^*) \cdot \frac{q_2(x_1^*)}{q_2(x_1^*)}
$$

is necessary and sufficient for $x_2^* > x_1^*$. Division by $U(x_1^*)$ together with (31) completes the proof of (53). The sufficient conditions follow by the same argument as in Theorem 2. QED.

Note that the additional condition (54) simply requires that the cost increase dominates the probability decrease.